Limits and their properties

Solve
1. use graph
2. plug in
3. symplify/cancel
4. rationalize
5. l'hopital's

Asymptotes
- when $y \to \infty$ - vertical asymptote
- when $x \to \infty$ - horizontal asymptote

- holes → when a factor cancels from num/denom
- VA → when denom = 0
- HA → when num = 0

Continuity
1. $f(c)$ is defined
2. $\lim_{x \to c} f(x)$ exists ✩ if lim right = lim left
3. $f(c) = \lim_{x \to c} f(x)$

Intermediate value theorum
- if continuous on $[a,b]$
  - ↳ a point (k) between $[a,b]$ exists

✩ $\lim_{x \to 0} \frac{\sin x}{x} = 1$

✩ $\lim_{x \to 0} \frac{1-\cos x}{x} = 0$

Differentiation

fermot's
$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+h) - f(x)}{h}$

differentiability
- must be continuous
- no sharp corners
- no vertical tangent

extreme value theorem
- if cont. over $[a,b]$, f has a max and a min over the interval

rolle's theorem
- if $f(a) = f(b)$, has to be a pt $f'(c) = 0$ (must turn around at one pt)

mean value theorem
- If cont over $[a,b]$, has to be a pt. c where $f'(c) = \frac{f(b) - f(a)}{b - a}$

extrema
✩ use [,] except for holes/asym
- increasing → $f'(x) > 0$
- decreasing → $f'(x) < 0$
- Check end points
- check where $f'(x) = 0$ or DNE (critical num)
  - max → + to -
  - min → - to +

concavity
✩ always para (,)
- conc up → $f''(x) > 0$
- conc down → $f''(x) < 0$
- poi → $f''(x) = 0$
  - ✩↳ $f''(x)$ changes sign ✩

BASIC DIFFERENTIATION RULES FOR ELEMENTARY FUNCTIONS

1. $\frac{d}{dx}[cu] = cu'$	2. $\frac{d}{dx}[u \pm v] = u' \pm v'$	3. $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$	5. $\frac{d}{dx}[c] = 0$	6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$	8. $\frac{d}{dx}[\lvert u\rvert] = \frac{u}{\lvert u\rvert}(u'), \quad u \neq 0$	9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$	11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$	12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$
13. $\frac{d}{dx}[\sin u] = (\cos u)u'$	14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$	15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$
16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$	17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$	18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$
19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$	20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$	21. $\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$
22. $\frac{d}{dx}[\text{arccot } u] = \frac{-u'}{1+u^2}$	23. $\frac{d}{dx}[\text{arcsec } u] = \frac{u'}{\lvert u\rvert\sqrt{u^2-1}}$	24. $\frac{d}{dx}[\text{arccsc } u] = \frac{-u'}{\lvert u\rvert\sqrt{u^2-1}}$

Integration

✩ even function → symetrical over y

✩ odd function → symmetrical over orgin

integrate (product)
1. distribute
2. u-sub
3. by parts
   - tabular
   - $\int u\,dv = uv - \int v\,du$ ✩ $u = \ln x$

BASIC INTEGRATION RULES ($a > 0$)

1. $\int kf(u)\,du = k\int f(u)\,du$	2. $\int [f(u) \pm g(u)]\,du = \int f(u)\,du \pm \int g(u)\,du$
3. $\int du = u + C$	4. $\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
5. $\int \frac{du}{u} = \ln\lvert u\rvert + C$	6. $\int e^u\,du = e^u + C$
7. $\int a^u\,du = \left(\frac{1}{\ln a}\right)a^u + C$	8. $\int \sin u\,du = -\cos u + C$
9. $\int \cos u\,du = \sin u + C$	10. $\int \tan u\,du = -\ln\lvert\cos u\rvert + C$
11. $\int \cot u\,du = \ln\lvert\sin u\rvert + C$	12. $\int \sec u\,du = \ln\lvert\sec u + \tan u\rvert + C$
13. $\int \csc u\,du = -\ln\lvert\csc u + \cot u\rvert + C$	14. $\int \sec^2 u\,du = \tan u + C$
15. $\int \csc^2 u\,du = -\cot u + C$	16. $\int \sec u \tan u\,du = \sec u + C$
17. $\int \csc u \cot u\,du = -\csc u + C$	18. $\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\frac{u}{a} + C$
19. $\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\frac{u}{a} + C$	20. $\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\text{arcsec}\frac{\lvert u\rvert}{a} + C$

integrate (quotient)
1. long div (if deg num > deg denom
2. u-sub
3. partial fractions
4. inverse trig
5. by parts

Riemann sums
- left end pt
- mid pt
- right end pt
- trapezoidal $\left(\frac{1}{2}(a+b)h\right)$

fundamental theorem of calculus
- $\int_a^b f(x)\,dx = F(b) - F(a)$

✩ speeding up when $f'(x)$ and $f''(x)$ have same sign

area

$\int_a^b r\,dx - \int_a^b (\text{hole})\,dx$

☆ in respect to y-axis
↳ solve for x
↳ switch to dy

$\frac{\pi r^2}{2}$

revolution (disc)

$\pi\int_a^b r^2\,dx - \pi\int_a^b (\text{hole})^2\,dx$

cross section

- square $\rightarrow \int_a^b s^2\,dx$
- semicircle $\rightarrow \frac{\pi}{2}\int_a^b r$
- issosiles $\rightarrow \int$

Arc length

- $\int_a^b \sqrt{1+(f'(x))^2}$

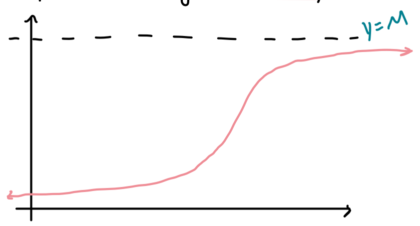
Differential Equations

Euler's method

- $f(x)$

x	y	
1	0	
2	3	$0 + h(f'(x,y))$
3	6	$3 + h(f'(x,y))$

Exponential growth/decay



$\frac{dy}{dx} = M\left(1-\frac{P}{M}\right)$

- M → maximum carrying capasity
- increasing fastest at $M/2$

improper integrals

- replace the ∞ with "b" and take $\lim_{b\to\infty}(\ )$
- use for non-existant points too
- or split the integral

Series

n^{th} term test

- $\lim_{n\to\infty}(f(x)) \neq 0 \rightarrow$ diverges

geo series

☆ sum $= \frac{a_1}{1-r}$

- $a(r)^n$

↳ if $r<1$ conv
↳ if $r\geq 1$ div

p-series

- $\left(\frac{1}{n^p}\right)$

↳ if $p\leq 1$ div
↳ if $p>1$ conv

alternating series test

- $(-1)^n a_n$

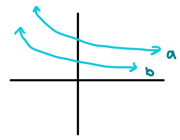
↳ ① $a_{n+1} \leq a_n$

② $\lim_{n\to\infty} a_n = 0$

integral test

- $\int_1^\infty a_n \rightarrow$ tells you if converge / diverge

direct comparison test



- if a converges → b converges
- if b diverges → a diverges

limit comparison test

- $\lim_{n\to\infty}\left|\frac{a_n}{b_n}\right| =$ positive constant (not 0) → same behavior as b_n

☆ b_n → comparitive function

ratio test

- $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$ $<1 \rightarrow$ converges
  $>1 \rightarrow$ diverges
  $=1 \rightarrow$ inconclusive

radius of convergance

- $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$

root test

- $\lim_{n\to\infty}\sqrt[n]{|a_n|} < 1$ converges
- $\lim_{n\to\infty}\sqrt[n]{|a_n|} > 1$ diverges
- $\lim_{n\to\infty}\sqrt[n]{|a_n|} = 1$ inconclusive

Taylor Series

- $f(c) + f'(c)(x-c) + \frac{f''(c)(x-c)^2}{2!} + \dots \frac{f^n(c)(x-c)^n}{n!} \dots$

Alternating series error

- the next term

Larrange error

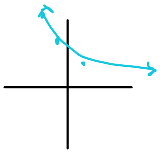
- $R_n(x) = \frac{f^{n+1}(z)_{max}(x-c)^{n+1}}{(n+1)!}$

known series

- $\frac{1}{1-x} = 1 + x + x^2 + x^3 \dots x^n$
- $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \frac{x^n}{n!} \dots$
- $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \dots \frac{(-1)^n x^{2n+1}}{(2n+1)!} \dots$
- $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \dots \frac{(-1)^n x^{2n}}{(2n+1)!}$

Parametrics

t	x	y
-3	-3	12
-2	-1	8
-1	1	4



☆ check domain

derivatives

- $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$
- $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$

integration

- arc length (distance traveled)

↳ $\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

velocity

- $\|v(t)\| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

☆ arc length = $\int v(t)\, dt$

Polar

equations

- (r, θ)
- $y = r\sin\theta$
- $x = r\cos\theta$
- $r^2 = x^2 + y^2$
- $\tan\theta = \frac{y}{x}$

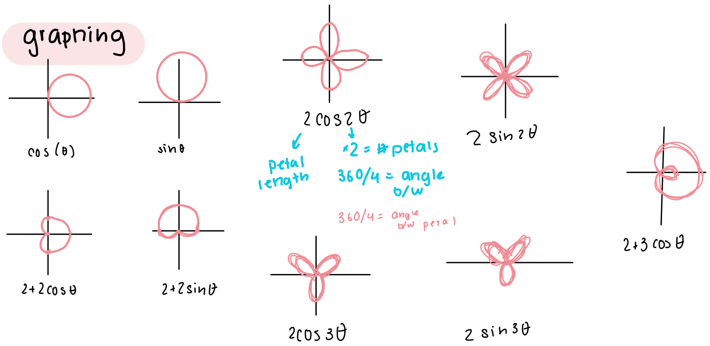
☆ slope of tangent =

$\frac{dy}{dx} = \frac{r\cos\theta + \sin\theta \frac{dr}{d\theta}}{-r\sin\theta + \cos\theta \frac{dr}{d\theta}}$

area

$\frac{1}{2}\int_a^b (f(\theta))^2\, d\theta$

graphing



Vectors

position $\langle x(t), y(t) \rangle$

displacement $\left\langle \int_a^b x'(t), \int_a^b y'(t) \right\rangle$

new location = $\left\langle x(a) + \int_a^b x'(t) \ldots, \right\rangle$

velocity $\langle x'(t), y'(t) \rangle$

acceleration $\langle x''(t), y''(t) \rangle$

distance traveled $\int_a^b (\text{speed})\, dt$

speed $\underbrace{\sqrt{(x'(t))^2 + (y'(t))^2}}_{\text{velocity}}$